THE TRACTARIAN THEORY OF DESCRIPTIONS

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Abstract:

The purpose of this paper is to offer an adequate interpretation of the Tractarian account of definite descriptions, thus contributing to a better understanding of the early Wittgenstein's philosophy. It is argued that the Tractarian account of the referential role played by definite descriptions in subject-position, despite the logical modifications imposed by Wittgenstein, is ultimately equivalent to the Russellian Theory of Descriptions. In order to justify this claim, the logical alterations imposed to the Theory of Descriptions by the Tractarian accounts of identity, generality and axiomatic method are considered in detail.

I - INTRODUCTION

It is well known that the early Wittgenstein admired Russell's Theory of Descriptions by the time the Tractarian philosophy was fermenting in his mind. Wittgenstein's admiration was so great that he apparently adopted the Theory in the Tractarian system. Nevertheless, he also seems to have modified the Russellian account. This may be confirmed by the important remark in a letter Wittgenstein wrote to Russell in 1913:

> The only other thing I want to say is that your Theory of Descriptions is *quite* CERTAINLY correct, even if the individual primitive signs in it are quite different from what you believe" (LRKM, p. 41, 43-4, emphasis Wittgenstein's).

The passage clearly shows that in 1913 Wittgenstein adopted the Russellian Theory of Descriptions, although he thought the Theory was in need of qualification.

Now it seems a reasonable conjecture that Wittgenstein did not change his mind by the time he wrote the *Tractatus* (see, for instance, TLP 2.0201; 4.0031). If this is correct, Wittgenstein not only adhered to the Theory of Descriptions, but also modified it in the *Tractatus*. What is more, the

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1913 remark seems to express, though obscurely, the essentials of the modification. By this I mean that the 1913 reference to primitive simple signs suggests that most of the modifications are connected with an alteration of the general framework within which the Theory is to be viewed.

The purpose of this paper is to clear up the meaning of the 1913 remark. In other words, I shall try to unveil in what sense the early Wittgenstein's solution to the problem of the referential mechanisms involved by definite descriptions in subject-position is a variant of the Russellian Theory of Descriptions. In order to do this, I shall take the following steps. First, I shall expound the basic principles of the Tractarian system and their possible consequences concerning the way definite descriptions refer. It will be shown that this involves a discussion of the alterations imposed to the Theory of Descriptions by the Tractarian accounts of identity, generality and axiomatic method. Second, I shall analyze to what extent the Tractarian account of identity logically alters the Russellian insight. Third, I shall try to offer an adequate interpretation of the obscure Tractarian account of generality. Fourth, on the basis of such an interpretation, I shall analyze to what extent the Tractarian account logically alters the Russellian insight. Fifth, I shall discuss the logical consequences, for Russell's account, of the Tractarian rejection of the axiomatic method. Sixth and finally, I shall list the main conclusions we may extract from the above discussion.

Although the *Tractatus* is cast among the most significant philosophical works in our century, it is notoriously obscure. I expect that this paper will contribute to a better understanding of the Tractarian system.

II - THE ALTERATIONS IMPOSED ON THE THEORY OF DESCRIPTIONS BY THE GENERAL PRINCIPLES OF THE *TRACTATUS*

According to the Tractarian view, any descriptive language requires the existence of a fully analyzed language. The latter is a transcendental condition of possibility of the former. The fully analyzed language is composed of elementary propositions which are combinations of primitive simple signs. It is a logically perfect language. But the logical connectives belonging to such a language are not Frege's and Russell's 'primitive signs'. In fact, 'v', '⊃', etc., are interdefinable, and this is enough to show that they are not primitive signs (TLP 5.42). In addition, the logical form of the elementary propositions is unknown, and it need not have the slightest similarity with the subject- predicate form of ordinary language, or Frege's *Begriffsschrift*", or Russell's formal language of *Principia*. We are unable to give the composition of elementary propositions, although we may have some concept of elementary propositions independently of their logical form (TLP 5.55; 5.555). It is the application of logic that decides what elementary propositions there are (TLP 5.557). The only thing we are able to say is that the definiteness of sense of any descriptive language transcendentally requires a one to one correspondence between each proposition of the descriptive language and a set of elementary propositions belonging to the fully analyzed language. Thus, it is true that the Russellian paraphrase shows the real logical form of a proposition containing a definite description in subject-position. But the paraphrase does not represent the final stage in the Tractarian analysis. Actually, the application of the latter would lead to the elementary propositions of the logically perfect language. With some minor alterations, Russell's formal language of *Principia* would be only an intermediary stage of the complete Wittgensteinian analysis.

The above seems to be a reasonable interpretation of Wittgenstein's claim that the primitive signs in the Theory of Descriptions are quite different from what Russell believes them to be. But the result is that not only Russell, but also Wittgenstein does not know what are the primitive simple signs. Thus, Wittgenstein's logically perfect language is only an implicit condition of possibility of any descriptive language, whereas Russell's formal language is clearly explicit in *Principia*. To put it more clearly, suppose a declarative sentence containing a definite description in subject-position. It is well-known that Russell's Theory of Descriptions exhibits the full analysis of the sentence by means of the formal language of *Principia*. By contrast, the Tractarian Theory assumes that the full analysis of the sentence is a transcendental condition of possibility of the sentence's making sense. In other words, the Tractarian full analysis is only postulated, not exhibited. And its result might be an articulation of elementary propositions which would be completely different from the Russellian paraphrase. This may be an important difference between the two Theories.

In order to clear the issue, call the Tractarian fully analyzed language *W-language* and Russell's formal language in *Principia R-language*. The former does not seem to correspond to the latter. The R-language is explicitly elaborated, whereas the W-language is only postulated and constitutes the transcendental condition of possibility of any descriptive language.

The R-language was obtained by analyzing ordinary language and is intended to give the correct logical form of the propositions of ordinary language. But then the R-language may be seen either as an intermediary stage in the analysis going from ordinary language to the W-language or as a descriptive language of which the W-language is the transcendental condition of possibility. Now if both the R- and W- language had the same expressive power and were logically equivalent, one might paraphrase Wittgenstein's aphorism 5.5563 and say: "In fact, all the propositions of our R-language, just as they stand, are in perfect logical order". Thus, no adaptations would have to be made in *Principia* in order to make valid its correspondence with the W- language. If this were possible, the fact that the primitive simple signs of the W-language are not the ones belonging to the R-language does not clash with the basic correctness of the logical order of the R- language. And here we would be back to Wittgenstein's 1913 remark, which tells us that, although the primitive simple signs are not what Russell believes them to be, the Theory of Descriptions is *quite certainly* correct.

As it will be shown later, the R- and the W- language have the same expressive power and the conjecture that both languages are logically equivalent is a good one. This may be shown by the discussion of other alterations Wittgenstein imposes to the logical framework of the R-language, which also seem to have an effect on the Theory of Descriptions. The most important of such alterations are: i) the rejection of the identity sign (TLP 4.243; 5.5301; 5.5303; 5.5321); ii) the strange dissociation of generality from truth-functions (id.: 5; 5.502; 5.51; 5.52; 5.521-5.526); iv) the rejection of the axiomatic method (6.1203; 6.1265; 6.1271). Now the question is: how far these alterations go from a purely logical standpoint? In other words, do the modifications listed yield a Tractarian Theory of Descriptions which is logically different from the Russellian one? Let us analyze the logical consequences of each of the alterations above listed.

III - THE ALTERATIONS IMPOSED ON THE THEORY OF DESCRIPTIONS BY THE TRACTARIAN ACCOUNT OF IDENTITY

Consider the problem of Wittgenstein's rejection of the identity sign in his analysis of descriptions in subject-position. Suppose the sentence to be analyzed is

(1) $f\{(\iota x)(Fx)\}$.

The Russellian analysis yields

(2) $f\{(\iota x)(Fx)\} = (\exists x)(y)((Fy \equiv y=x) \& fx)$ Df,

whereas the Wittgensteinian analysis yields

(3) $f\{(\iota x)(Fx)\} = (\exists x)(Fx \& \sim(\exists x)(\exists y)(Fx\&Fy) \& Bx)$ Df.

In (3), *(∃x)(Fx & ~(∃x)(∃y)(Fx&Fy)* means that only one *x* satisfies the function *Fx* (TLP 5.5321). The possibility of this replacement shows that the identity-sign is not an essential constituent of conceptual notation (TLP 5.533).

Now this modification does not make the Tractarian system radically different from the Russellian system of *Principia*. As a matter of fact, even Wittgenstein's treatment of identity in the *Tractatus* may yield a system which is logically equivalent to *Principia*. But this can be made only with some qualifications, because the account of (3) according to the methods and processes established by *Principia Mathematica* would yield disappointing results. For example, a contradiction may be derived from (3) in the following way:

1. (Ex)(Fx & -(Ex)(Ey)(Fx & Fy) & Bx) [(3) taken as a premise].
2. Fa & -(Ex)(Ey)(Fx & Fy) & Ba [1, existential instantiation].
3. Fa & (x)(y)-(Fx & Fy) & Ba [2, equivalence].
4. Fa & (y)-(Fa & Fy) & Ba [3, universal instantiation].
5. Fa & -(Fa & Fa) & Ba [4, universal instantiation].
6. Fa & -Fa & Ba [5, equivalence].
7. Fa & -Fa [6, simplification].

Thus, if treated according to Russellian patterns, the Wittgensteinian analysis of definite descriptions in subject-position seems to collapse into contradiction. Could it be that, contrary to Wittgenstein's thought in the *Tractatus*, identity is such an essential sign in logic that we cannot dispense with it?

Fortunately for Wittgenstein's account, this does not seem to be so. In fact, in an interesting 1956 paper, *Identity, Variables and Impredicative Definitions*, Hintikka follows the Tractarian suggestion and succeeds in constructing a logic without identity. Hintikka's main point is that variables can be used in a twofold way. First, we may use them in a way such that coincidences of the values of different variables are not excluded. This is what Hintikka calls the *inclusive interpretation* of variables (1956: 226). Second, we may use them in a way such that coincidences of the values of different variables are excluded. This is what Hintikka calls the *exclusive interpretation* of variables (*ibid.*). The latter may be either weakly or strongly exclusive (1956: 230).

Now Hintikka suggests that Wittgenstein adopts the weakly exclusive interpretation of variables in the *Tractatus* (1956: 228; 230). In support of his claim, Hintikka quotes the aphorisms 5.53-5.5352, where Wittgenstein is mainly discussing identity. Hintikka also proves that everything expressible by the inclusive quantifiers plus identity may be expressed by the weakly exclusive quantifiers without a sign for identity (1956: 235). Thus, he thinks that the Tractarian claim that identity is not an essential constituent of logical notation is correct (*ibid.*).

I think Hintikka is undoubtedly right in his claim. In fact, most of the Tractarian discussion on identity supports a weakly exclusive interpretation of variables. What is more, there is a passage in the *Notebooks* that may be quoted in support of such an interpretation of variables by Wittgenstein. He writes:

> I believe that it would be possible wholly to exclude the sign of identity from our notation and always to indicate identity merely by the identity of the signs (and conversely). In that case, of course, $\phi(a, a)$ would not be a special case of $(x, y) . \phi(x, y)$, and ϕa would not be a special case of $(\exists x, y) . \phi x . \phi y$. But then instead of $\phi x . \phi y \supset_{x,y} x = y$ one could simply write $\sim(\exists x,y) . \phi x . \phi y$. (NB p. 34, 29th November 1914).

Here, it is argued that $\phi(a, a)$ is not an instance of $(x)(y)\phi xy$: this would be possible only if the use of variables is exclusive.

But then we may lay down a rule for transforming an expression belonging to the W-language into an expression belonging to the R-language and vice versa. For example, suppose *(Ex)* and *(Ux)* stand for a weakly exclusive interpretation respectively of the existentially and the universally quantified variables. In this case, the Tractarian expression *(Ux)(Uy)Fxy* would involve the requirement that *x* should be different from *y*. Thus, the mere addition of the clause $x \neq y$ transforms the Tractarian expression into a Russellian one: *(Ux)(Uy)Fxy* is equivalent to $(x)(y)(Fxy \; \& \; x \neq y)$, or, with existential quantifiers, to $\sim(\exists x)(\exists y)(x=y \vee \sim Fxy)$.

The above procedure is generalized by Hintikka. He formulates translation rules by means of which the exclusive quantifiers may be paraphrased in terms of $(\exists x)$ and (x) (1956: 231). Hintikka's transformation rules seem to provide an adequate method for translating Tractarian expressions into Russellian ones and vice versa.[2] If this is correct, (3) should be expressed by weakly exclusive variables, thus yielding:

(4) (Ex)(Fx & Bx & ~(Ex)(Ey)(Fx & Fy)).

If one now applies Hintikka's transformation rules to (4), one obtains

(5) $(\exists x)(Fx \;\&\; Bx \;\&\; \sim(\exists x)(\exists y)(x \neq y \;\&\; (Fx \;\&\; Fy)))$.

Although not with the help of explicit Wittgensteinian rules for dealing with truth-tables involving quantification, it may be easily proved that (5) is equivalent to the Russellian paraphrase, namely

(6) $(\exists x)(Fx \;\&\; (y)(Fy \supset y=x) \;\&\; Bx)$.[3]

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[2] As to an expression like *(x)(x = x)*, it contains what has to be eliminated by means of Hintikka's transformation rules. The problem is solved by recalling that, in Russell's system, any well-formed formula, say *A*, is equivalent to the onjunction *A & T*, where *T* may stand for a tautology in which the identity-sign is absent. Thus, *(x)(x =x)* may be replaced by its equivalent *(x)((x = x) & T)*. In turn, the latter may be replaced by its Tractarian equivalent, namely *(Ux)T*. As a result, all the usual tautologies involving the identity-sign would be replaced by tautologies in which this sign would be absent. Although this procedure does not involve any logical error, it looks rather clumsy. Hintikka's rules may be supplemented by the Wittgensteinian rules for dealing with expressions involving constants. Thus, *F(a, a)* (Tractarian and Russellian expression) may be rendered as *F(a, b) & a = b* (Russellian expression only); *F(a, b)* (Tractarian and Russellian) may be rendered as $F(a, b) \;\&\; a \neq b$ (Russellian expression only). See TLP 5.531.

[3] For example, a proof that (5) entails (6) would run:

1. $(\exists x)(Fx \;\&\; Bx \;\&\; \sim(\exists x)(\exists y)(x \neq y \;\&\; (Fx \;\&\; Fy)))$ [premise]
2. $Fa \;\&\; Ba \;\&\; \sim(\exists x)(\exists y)(x \neq y \;\&\; (Fx \;\&\; Fy))$ [1, EI]
3. Fa [2, Simp]
4. Ba [2, Simp]
5. $\sim(\exists x)(\exists y)(x \neq y \;\&\; (Fx \;\&\; Fy))$ [2, Simp]

Therefore, the Russellian account of (3) is inadequate, because it misleadingly assumes that the Wittgensteinian variable is to be interpreted in the same way as the variables of "Principia Mathematica". The disappointing results are not the true ones. Besides, the equivalences obtained within Hintikka's composite formal system in fact confirm Wittgenstein's claim that the identity-sign is not an essential constituent of logical notation (Hintikka 1956: 230, fn. 11; 235). The main result of the above discussion is that the Tractarian system has the same expressive power of *Principia*. And this fact increases the possibility that both systems are logically equivalent. Therefore, as far as identity is concerned, the Wittgensteinian modification of the general framework of the Theory of Descriptions does not alter its basic logical aspects. And an alleged divergence between the Tractarian Theory of Descriptions and the Russellian one reveals to be a possible source for the logical equivalence between the two Theories.

IV - THE TRACTARIAN ACCOUNT OF GENERALITY

Consider now the difficult question of Wittgenstein's obscure and apparently contradictory account of generality in the *Tractatus*. The general propositions seem to be at the same time logical prototypes (TLP 5.501; 5.51-52) and truth-functions of elementary propositions(TLP 5; 5.521-26). In what follows, I shall argue that, despite the appearance of contradiction, this may be explained in a coherent way. In addition, I believe one may propose at least one interpretation of the generalized propositions in a way such that the Tractarian system may be proved equivalent to the system of *Principia*. The interpretation is based on the one found in Russell's *Introduction* to the *Tractatus*.

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6. (x)(y)~(x ≠ y & (Fx & Fy)) [5, Equiv]
7. (x)(y)(Fx ⊃ (Fy ⊃ y = x)) [6, Equiv]
8. Fx ⊃ (Fy ⊃ y = x) [7, UI]
9. Fa ⊃ (Fy ⊃ y = a) [9, EI]
10. Fy ⊃ y = a [3,9 MP]
11. (y)(Fy ⊃ y = a) [10, UG]
12. Fa & Ba & (y)(Fy ⊃ y = a) [3, 4, 11 Conj]
13. (∃x)(Fx & Bx & (y)(Fy ⊃ y = a)) [12, EG]
14. (5) ⊃ (∃x)(Fx & Bx & (y)(Fy ⊃ y = a)) [1-13, CP].

Here, *EI* and *EG* stand for *existencial instantiation* and *existential generalization* respectively; *UI* and *UG* for *universal instantiation* and *universal generalization* respectively; *Simp*, *Equiv*, *MP*, *Conj*, and *CP* stand for *simplification*, *logical equivalence*, *modus ponens*, *conjunction*, and *conditional proof* respectively.That the converse implication also holds may be proved along the linhes of a *reductio ad absurdum*.

The fact that a general proposition is a logical prototype may be explained in the following way. We know that: i) an 'expression' is any part of a proposition that characterizes its sense (TLP 3.31); ii) a proposition is an expression (id.). Given these definitions, suppose we turn a constituent of a proposition into a variable. For example, suppose we turn the argument *John* in the proposition *John sleeps* into the variable *x*, thus obtaining *x sleeps*. Wittgenstein calls this a *variable proposition* (cf. 3.315), but according to aphorism 3.314, *x sleeps* may be also interpreted as a propositional variable. In this case, *x sleeps* specifies a class of propositions all of which are its values. But here the class specified depends on the conventional meaning of the expression *sleeps*. If we now turn the latter into an adequate variable, say *Yx*, we shall still obtain a propositional variable which determines a class of propositions of the same kind as the above mentioned. The class determined by *Yx* does not depend on any conventional meaning, but solely on the nature of the proposition formerly expressed by 'John sleeps' (cf. 1922: 3.315). In this case, *Yx* corresponds to a *logical prototype* (*ibid*.).

Now the proposition 'John sleeps' also allows the construction of quantified propositional variables, such as *(∃x)Yx*, *(x)Yx* and so on. These propositional variables also correspond to logical prototypes. The peculiarity of the generalized propositions is that they both indicate logical prototypes and give prominence to constants (TLP 5.522). In the above examples, the logical form is the constant to which prominence is given.

If this is correct, then the construction of generalized propositions through the procedure of replacing expressions by variables does not involve the appeal to truth-functions. This coheres with the Tractarian claim that propositions comprise all that follows from the totality of all elementary propositions (TLP 4.52). For the construction of logical prototypes follows from the totality of all elementary propositions. That is why Wittgenstein also claims that, in a certain sense, all propositions may be taken as generalizations of elementary propositions (*ibid*.).

The above interpretation explains Wittgenstein's claim that he dissociates the concept 'all' from truth-functions. But the fact that generalized propositions are also truth-functions of elementary propositions remains to be explained. This is connected with the Tractarian claim that the logical product and logical sum are notions *embedded* in the propositions *(x)Fx* and *(∃x)Fx* (TLP 5.521). Wittgenstein claims that the analysis of a proposition must bring us to elementary propositions (TLP 4221) and that a proposition is a truth- function of elementary propositions (TLP 5). What is more, elementary propositions themselves contain all logical operations (TLP 5.47), and this includes the quantifiers. These principles are general enough to be applied to generalized propositions. But then these propositions will also reveal themselves to be truth- functions of elementary propositions. Thus, although a generalized proposition may be constructed independently of truth-functions, the valuation of the generalized proposition will have to take us back to truth-functions. Now the question is: in what sense? The answer to this may be given as follows:

i) Once one obtains a logical prototype through the procedure of replacing expressions in a proposition by the corresponding variables, one stipulates the propositions which are the values of the propositional variable expressed by the logical prototype. The stipulation is made by means of the common logical form of the propositions stipulated. This is a purely syntactic procedure and constitutes the basis for establishing a logical equivalence between the prototype and a logical articulation of the propositions specified by the prototype (TLP 3.316-7). This seems to be similar to Russell's interpretation in his "Introduction" to the *Tractatus* (TLP xv). A similar interpretation is found in Fogelin (1976: 57).

ii) Once the stipulation is made, the truth- value of the logical prototype will be calculated by means of the truth-values of the propositions which are the values of the propositional variable expressed by the prototype. This coheres with the Tractarian claim that

> If we are given a proposition, then with it we are also given the results of all truth- operations that have it as their base (TLP 5.442).

Thus, in the case of a proposition involving, say, the universal quantifier, the logical prototype obtained will be true if and only if all the propositions belonging to the set specified by the prototype are true (cf. TLP 5.52). Therefore, the proposition *(x)Fx* will be equivalent to the logical product of the propositions belonging to the set specified by the prototype. As for a proposition involving the existential quantifier, the logical prototype obtained will be true if and only if at least one of the propositions belonging to the set specified by the prototype is true. Therefore, the proposition *(∃x)Fx* will be equivalent to the logical sum of the propositions belonging to the set specified by the prototype. This situation takes us from the prototype to truth-functions. For the generalized propositions, although constructed on the basis of the symbolism alone and independently of truth-functions, are ultimately equivalent to logical conjunctions or disjunctions of the propositions belonging to the sets specified by the prototypes obtained. The propositions belonging to the sets specified by the prototypes may in turn be fully analyzed into elementary propositions. Thus, a generalized proposition may be fully analyzed in a way such that it specifies a set of elementary propositions and is a truth-function of a logical sum or a logical product of the elementary propositions involved. This would explain Wittgenstein's claim that the logical product and the logical sum are embedded in generality (TLP 5521). Simultaneously, this would reveal that Russell's suggestion that Wittgenstein derives general propositions from conjunctions and disjunctions (TLP xvi) is not entirely correct. Actually, Wittgenstein needs conjunctions or disjunctions in order to assign a truth-value to a general proposition, not to construct it: the logical prototype is obtained by merely introducing the adequate variables.

Against the above interpretation, one might argue in the following way. As a logical prototype, the generalized proposition contains variables. But then its final analysis will still contain variables. Thus, it will always leave something undetermined (TLP 3.24). In other words, the analysis of the generalized proposition will not take us directly to articulations of elementary propositions containing primitive simple signs, but only to articulations of propositional variables containing variables which stand for primitive simple signs. Therefore, the indeterminateness of the generalized proposition would make it impossible for the proposition to be a truth- function of elementary propositions.

I would reply to this as follows. The indeterminateness notwithstanding, the generalized proposition clearly specifies a set of propositions and is a truth-function of the propositions belonging to the set. True, the final stage of the analysis of the generalized proposition would take us to propositional variables which would contain variables for simple signs. But the resulting propositional variables would still specify sets of elementary propositions. Thus, the generalized proposition may be also seen as a truth-function of elementary propositions. What is more, this squares with the Tractarian general principle that a proposition is a truth-function of elementary propositions (TLP 5). As a result, the generalized proposition reveals to be perfectly capable of describing the world.

This may also be inferred from aphorism 5.526, where Wittgenstein states that we can describe the world completely by means of fully generalized propositions. For a non-fully generalized proposition is simply a stage in the construction of a fully generalized one. Thus, a proposition like $(\exists x)(\exists Y)Yx$ differs from $(\exists x)Fx$ in that the set of propositions specified by the former is wider than the one specified by the latter. But the latter also describes a situation and as such may be compared with reality. As for indeterminateness, compare a proposition like *(∃x)Fx* with *(∃x)Fx & x=a*. The former is more indeterminate than the latter. Despite this, the former describes the world (in the Tractarian sense) as accurately as the latter if both are true. As a matter of fact, if *(∃x)Fx* is a true Tractarian description of the world, it depicts an existing state of affairs and shows the logical structure of the state of affairs within the logical space, thus mirroring the essence of the world; if *(∃x)Fx & x=a* is true, it adds to this an extra piece of information, namely *x=a*, which is a matter of detail and does not add anything relevant to the Tractarian description of the world made by *(∃x)Fx*. The essence of the world is logic. In order to describe the world in the Tractarian way, we only need to know that there are objects in the world, not which objects there are. Thus, although our customary mode of expression is rather made by means of *(∃x)Fx & x=a*, there is nothing wrong with the indeterminateness of *(∃x)Fx* from the standpoint of a true Tractarian description of the world.

V - THE ALTERATIONS IMPOSED ON THE THEORY OF DESCRIPTIONS BY THE TRACTARIAN ACCOUNT OF GENERALITY

If the above interpretation is correct, we now have to see how generality works in propositions containing definite descriptions in subject-position. This is connected with the Tractarian analysis of propositions containing a sign for a complex (TLP 3.24-3.261). In this respect, Wittgenstein makes the following claims.

i) The analysis of a proposition containing the sign for a complex is unique and not arbitrary (TLP 3.25; 3.3442). This principle may be applied to propositions containing definite descriptions, that is, signs for complexes, in subject-position. As a result, each of such propositions will have one and only one complete analysis.

ii) The complete analysis of a proposition containing the sign for a complex will involve propositions about constituents of the complex and propositions that describe the complex completely (TLP 2.201). This seems to be confirmed by the following aphorism:

> A proposition about a complex stands in an internal relation to a proposition about a constituent of the complex (TLP 3.24).

Although the aphorism is obscure, I shall argue that it reinforces 2.201. In order to justify my claim, I offer the following reasons: iia) the relation between the proposition about a complex and the proposition about the constituent of the complex is *internal*, and this means that the analysis of the former would take us necessarily to the latter; iib) the symbol for a complex is dissected into a simple symbol by means of a definition, thus signifying via the signs that serve to define it (the definitions point the way) (TLP 3.24; 3.261); iic) we assumed, as a reasonable conjecture, that Wittgenstein adhered to the Theory of Descriptions in the Tractarian period, and this entails that he would claim that the analysis of a proposition containing the sign for a complex would involve a proposition about a constituent of the complex; iic) my interpretation coheres with the Tractarian claim that the proposition about the complex will not be nonsensical, but simply false if there is no set of constituents linked in a way such that they satisfy the description of the complex (TLP 3.24). Now given that a complex can be given only by its description (*ibid*.), the above principle clearly applies to propositions containing definite descriptions in subject-position.

My interpretation also coheres with Wittgenstein's claim that the proposition containing the sign for a complex leaves something undetermined, just like it occurs with the generality sign (*ibid*.). But this does not entail that the proposition containing the sign for a complex will not be a truth- function of elementary propositions. In fact, the analysis of the proposition about the complex will involve generalized propositions and so the former will be a truth-function of elementary propositions in the same sense as the latter.

As a result, the Tractarian account of a proposition containing a definite description in subject-position, say *the F is P* may be summarized as follows. The analysis of *the F is P* will involve propositions about the constituents of *the F* and propositions that describe *The F* completely. These propositions are generalized, thus involving some indeterminateness. But this does not prevent such propositions of being truth-functions of elementary propositions. In fact, the final analysis of *the F is P* will be such that it will involve at least one generalized proposition of the form *there is an x such that...* The latter will specify a set of elementary propositions and will be a truth-function of the elementary propositions belonging to the set. If *the F* does not exist, all the elementary propositions belonging to the set specified will be false. Thus, *the F is P* will be false if *the F* does not exist. In turn, if *the F* does exist, then *the F is P* will be either true or false depending on the adequacy of the predicate *x is P* to the complex involved. Thus, *the F is P* is an authentic proposition that contains everything which is relevant for an accurate Tractarian description of the world.

If this is correct, then the Tractarian general propositions would be logically equivalent to the ones belonging to the classical first order predicate calculus with bound variables. So, as far as generality is concerned, Wittgenstein's TLP- and W- language are logically equivalent to Russell's R- language.

VI - THE ALTERATIONS IMPOSED ON THE THEORY OF DESCRIPTIONS BY THE REJECTION OF AXIOMATIC METHOD

We turn now to Wittgenstein's rejection of Russell's axiomatic method as adopted by *Principia*. The fact is that, although Wittgenstein accuses the axiomatic method of being misleading, the Tractarian system seems to be logically equivalent to the system of *Principia*. Actually, we already know that both systems have the same expressive power. This means that we can make a correspondence between the true propositions of both systems in a way such that: i) to every true proposition of the Tractarian system there corresponds one true proposition of the system of *Principia*; ii) to every true proposition of the R- language there corresponds one true proposition of the W-language.

Now this suggests that the above systems may be logically equivalent. Consider the propositional calculi involved by such systems. It is a well-known fact that every axiom and every theorem of the propositional calculus is a tautology and that every tautology is either an axiom or a theorem of the propositional calculus (Chauvineau 1962: 100-6). Ultimately, this means that: i) if a proposition of the propositional calculus is an axiom or theorem in the R-language, then it is a tautology in the TLP- language; ii) if a proposition of the propositional calculus is a tautology in the TLP-language, then it is an axiom or a theorem in the R-language. In other words, the R-language contains the propositional calculus in its axiomatic version, whereas the TLP- language contains the same calculus in its tautological version. Therefore, a proposition of the propositional calculus is an axiom or a theorem in the R-language if and only if it is a tautology in the TLP-language. Now consider the predicate calculi involved by the above systems. As already mentioned, the Tractarian general propositions seem to be logically equivalent to the ones belonging to the classical first order predicate calculus with bound variables.

Although the conjunction of these facts is not enough to prove that the TLP-language is logically equivalent to the R-language, it clearly suggests that both languages contain significant domains which are logically equivalent. This fact makes the claim that both languages are logically equivalent a good conjecture. Given that the TLP-language is logically equivalent to the W-language, the Tractarian system would reveal itself to be logically equivalent to the system of *Principia*. Even though the systems were not equivalent, their similarities seem to surmount greatly their discrepancies. The modifications imposed by Wittgenstein do not seem to alter the basic logical results obtained by Russell and the Tractarian Theory of Descriptions seems to be logically equivalent to the Russellian.

VII - FINAL REMARKS

As a whole, the analysis of the Wittgensteinian modifications shows that the Tractarian system would not probably entail any radical alteration of the Russellian Theory of Descriptions. This result puts a significant part of the Tractarian system on a par with the first order predicate calculus with bound variables of *Principia*. Of course, the decision to use one of these systems rather than the other is determined by practical reasons. And the preference given to the system of *Principia* speaks for itself.

As a result, the early Wittgenstein's solution to the problem of referential role played by definite descriptions in subject-position seems to be ultimately equivalent to Russell's, although the general formal framework to which the solution belongs is different. But this is true under the following qualifications: i) the equivalence mentioned occurs basically within the domain of propositional calculus and first-order predicate calculus; ii) although logically correct, the Tractarian claim that identity is dispensable is dogmatically stated, not proved; iii) the possibility of constructing truth-tables for expressions involving quantifiers is also dogmatically stated, not proved. But the more important aspect of Wittgenstein's solution is that the basic result of Russell's Theory is preserved: a proposition containing an empty description in subject-position is not nonsensical, but simply false (TLP 3.24; 5.473).

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